

Delay-Aware Uplink Fronthaul Allocation in Cloud Radio Access Networks

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Abstract

In cloud radio access networks (C-RANs), the baseband units and radio units of base stations are separated, which requires high-capacity fronthaul links connecting both parts. In this paper, we consider the delay-aware fronthaul allocation problem for C-RANs. The stochastic optimization problem is formulated as an infinite horizon average cost Markov decision process. To deal with the curse of dimensionality, we derive a closed-form approximate priority function and the associated error bound using perturbation analysis. Based on the closed-form approximate priority function, we propose a low-complexity delay-aware fronthaul allocation algorithm solving the per-stage optimization problem. The proposed solution is further shown to be asymptotically optimal for sufficiently small cross link path gains. Finally, the proposed fronthaul allocation algorithm is compared with various baselines through simulations, and it is shown that significant performance gain can be achieved.

Index Terms

cloud radio access networks, fronthaul link, delay optimization, perturbation analysis, Markov decision process

This work is supported in part by National Natural Science Foundation of China (No. 61261130585), Research Grants Council (RGC) of Hong Kong (No. N_HKUST605/13), and Hong Kong Scholars Program (No. 2012T50566).

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I. INTRODUCTION

The cloud radio access network (C-RAN) [1] provides a new architecture for 5G cellular systems. In C-RANs, the baseband processing of base stations is carried out in the cloud, i.e., a centralized base band unit (BBU), which launches joint signal processing with coordinated multi-point transmission (CoMP) and makes it possible to mitigate inter-cell interference. The separation of the BBU and the radio units (RUs) brings a new segment, i.e., fronthaul links, to connect both parts. The limited capacities of fronthaul links have a significant influence on the system performance of C-RANs.

There are several existing works on fronthaul links in C-RANs. Efficient signal quantization/compression for fronthaul links is designed to maximize the network throughput for the uplink and downlink in [2] and [3], respectively. In [4], fronthaul quantization and transmit power control are optimized jointly. In [5], energy-efficient CoMP is designed for downlink transmission considering fronthaul capacity. In [6], the capacities of fronthaul links are allocated under a sum capacity constraint to maximize the total throughput. In [7], the fronthaul links are reconfigured to apply appropriate transmission strategies in different parts according to both heterogeneous user profiles and dynamic traffic load patterns. However, these existing works have all focused on the physical layer performance without consideration of bursty data arrivals at the transmitters or of the delay requirement of the information flows. Since real-life applications (such as video streaming, web browsing or VoIP) are delay-sensitive, it is important to optimize the delay performance of C-RANs.

To take the queueing delay into consideration, the fronthaul allocation policy should be a function of both the channel state information (CSI) and the queue state information (QSI). This is because the CSI reveals the instantaneous transmission opportunities at the physical layer and the QSI reveals the urgency of the data flows. However, the associated optimization problem is very challenging. A systematic approach to the delay-aware optimization problem is through a Markov Decision Process (MDP). In general, the optimal control policy can be obtained by solving the well-known *Bellman equation*. Conventional solutions to the Bellman equation, such as brute-force value iteration or policy iteration [8], have huge complexity (i.e., the curse of

dimensionality), because solving the Bellman equation involves solving an exponentially large system of non-linear equations.

In this paper, we focus on minimizing the average delay by fronthaul allocation. There are two technical challenges associated with the fronthaul allocation optimization problem:

- **Challenges due to the Average Delay Consideration:** Unlike other works which optimize the physical layer throughput, the optimization involving average delay performance is fundamentally challenging. This is because the associated problem belongs to the class of *stochastic optimization* [9], which embraces both *information theory* (to model the physical layer dynamics) and *queueing theory* (to model the queue dynamics). A key obstacle to solving the associated Bellman equation is to obtain the priority function, and there is no easy and systematic solution in general [8].
- **Challenges due to the Coupled Queue Dynamics:** The queues of data flows are coupled together due to the mutual interference. The associated stochastic optimization problem is a K -dimensional MDP, where K is the number of data flows. This K -dimensional MDP leads to the curse of dimensionality with complexity exponential to K for solving the associated Bellman equation. It is highly nontrivial to obtain a low complexity solution for dynamic fronthaul allocation in C-RANs.

In this paper, we model the fronthaul allocation problem as an infinite horizon average cost MDP and propose a low-complexity delay-aware fronthaul allocation algorithm. To overcome the aforementioned technical challenges, we exploit the specific problem structure that the cross link path gain is usually weaker than the home cell path gain. Utilizing the *perturbation analysis* technique, we obtain a closed-form approximate priority function and the associated error bound. Based on that, we obtain a low-complexity delay-aware fronthaul allocation algorithm. The solution is shown to be asymptotically optimal for sufficiently small cross link path gains. Furthermore, the simulation results show that the proposed fronthaul allocation achieves significant delay performance gain over various baseline schemes.

The rest of this paper is organized as follows. In Section II, we establish the wireless access link, fronthaul link and cloud baseband processing models as well as the queue dynamics. In

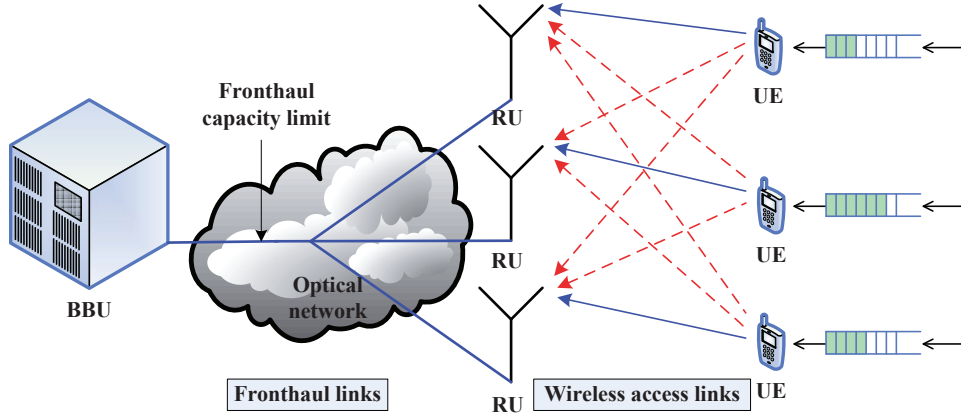


Figure 1. C-RAN topology

In Section III, we formulate the fronthaul allocation problem and derive the associated optimality conditions. In Section IV, we propose a low-complexity fronthaul allocation solution. Following this, the delay performance of the proposed algorithm is evaluated by simulation in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

In this section, we introduce the C-RAN topology and the associated models of the access link, the fronthaul link and the cloud baseband processing. Based on the models, we obtain the throughput and the dynamics of packet queues.

A. C-RAN Topology

We consider a C-RAN with K cells, each of which has an RU with a single antenna. In each cell, the data are transmitted from a single-antenna user equipment (UE) to the RU via wireless access links and then to the BBU via the fronthaul link over fiber/microwave, as shown in Fig. 1.

The time is slotted and the duration of each time slot is τ . The BBU collects necessary information and makes the resource allocation decisions periodically at the beginning of each time slot.

B. Wireless Access Link Model

The wireless access links are modeled as an interference channel. In the uplink, the UEs transmit signals to their corresponding RUs respectively, and in the meantime, cause interference to other RUs in the network. The signals received by the RUs are

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \quad (1)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_K)^T$ is a K -dimensional vector of the transmitted signals, in which x_k is transmitted by the k -th UE with power P , $\mathbf{y} = (y_1, y_2, \dots, y_K)^T$ is a K -dimensional vector of the signals received by the RUs, in which y_k is the signal received by the RU in the k -th cell, $\mathbf{H} = (H_{kj})_{K \times K}$, in which H_{kj} is the complex channel fading coefficient of the uplink transmission from the j -th UE to the RU in the k -th cell, $\mathbf{z} = (z_1, z_2, \dots, z_K)^T$ and $z_k \sim \mathcal{CN}(0, N_0)$ is the white Gaussian thermal noise with power N_0 .

Define $\mathbf{H}(t)$ as the *global CSI* for uplink access links at the t -th slot. We have the following assumption on $\mathbf{H}(t)$.

Assumption 1 (CSI Model): The CSI $\mathbf{H}(t)$ remains constant within a time slot and is i.i.d. over time slots. $H_{kj}(t)$ is independent over the indices k and j .¹ $H_{kj}(t)$ is composed of two parts, i.e., $H_{kj}(t) = \sqrt{L_{kj}}\tilde{H}_{kj}(t)$, where $\tilde{H}_{kj}(t)$ is the short-term fading coefficient which follows a complex Gaussian distribution with mean 0 and unit variance, and L_{kj} is the corresponding large-scale path gain, which is constant over the duration of the communication session. ■

C. Fronthaul Link Model

Denote $C_k(t)$ as the capacity allocated to the fronthaul link between the RU in the k -th cell and the BBU at the t -th slot. Let $\mathbf{C}(t) = (C_1(t), C_2(t), \dots, C_K(t))$ be the uplink fronthaul allocation. With limited-capacity fronthaul links, the signals transmitted between the RUs and the BBU have to be quantized. In the uplink, the RU in each cell underconverts its received

¹In C-RANs, the simultaneously transmitting UEs using the same resource block are located in different cells. Thus, the distances between the RUs and those between the UEs are always large enough to make the channel fading coefficients independent.

signal and sends the quantized signal to the BBU. Define $\hat{\mathbf{y}} = (\hat{y}_1, \hat{y}_2, \dots, \hat{y}_K)^T$, where \hat{y}_k is the quantized signal at the RU in the k -th cell. The signals are assumed to be quantized for each fronthaul link separately. The quantization leads to the distortion of signal, which can be treated as the quantization noise, denoted as $\mathbf{n} = (n_1, n_2, \dots, n_K)^T$, where n_k is the quantization noise over the k -th fronthaul link. The signals received by the BBU are expressed as

$$\hat{\mathbf{y}} = \mathbf{y} + \mathbf{n}. \quad (2)$$

The relationship between y_k and \hat{y}_k depends on the fronthaul capacity C_k according to the rate-distortion theory [10], which is given by $I(y_k : \hat{y}_k) \leq C_k$, where $I(y_k : \hat{y}_k)$ is the mutual information between y_k and \hat{y}_k . Let $\mathbf{N}(t) = (N_1(t), N_2(t), \dots, N_K(t))$, where $N_k(t)$ is the power of the quantization noise n_k at the t -th slot. The quantization noise power induced by the transmission over the k -th uplink fronthaul link at the t -th slot is given by [6]

$$N_k(t) = \frac{P \sum_{j=1}^K \|H_{kj}(t)\|^2 + N_0}{2^{C_k(t)} - 1}, \quad (3)$$

where $\|\bullet\|$ is the Euclidean norm.

D. Throughput with Cloud Baseband Processing

The BBU performs joint decoding for the received uplink signals, which benefits the system performance by joint cloud processing of the signals for different cells. The cloud baseband processing for uplink signals at the BBU is introduced in the following assumption.

Assumption 2 (Zero Forcing Joint Detection): Assume that ZF joint detection [11], [12] is adopted for the uplink in the cloud baseband processing to eliminate the inter-cell interference. The linear ZF receiver at the BBU can be represented by a matrix $\mathbf{S}(t) = (S_{kj}(t))_{K \times K}$ at the t -th slot, where $\mathbf{S}(t)$ is the inverse² of the channel matrix $\mathbf{H}(t)$, i.e., $\mathbf{S}(t) = \mathbf{H}(t)^{-1}$. ■

The uplink transmission model is described in Fig. 2. With the ZF joint detection at the BBU, the post-processing signal is

$$\mathbf{S}\hat{\mathbf{y}} = \mathbf{x} + \mathbf{S}(\mathbf{z} + \mathbf{q}). \quad (4)$$

²According to Assumption 1, the elements of $\mathbf{H}(t)$ are independent. Thus, $\text{rank}(\mathbf{H}(t)) = K, \forall t$ and the inverse of $\mathbf{H}(t)$ exists.

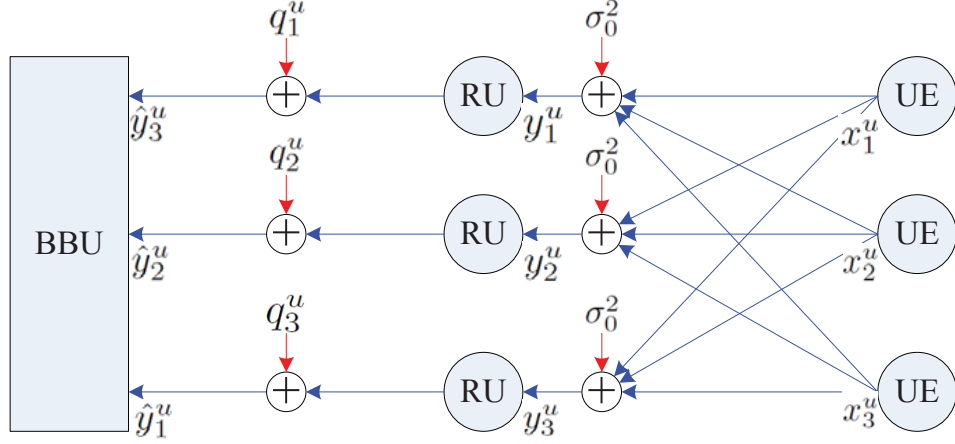


Figure 2. Uplink transmission model of a C-RAN

Considering both the thermal noise power N_0 and the quantization noise power $N(t)$, we obtain the uplink data rate for the i -th UE as

$$R_k(\mathbf{H}(t), \mathbf{C}(t)) = \log_2 \left(1 + \frac{P}{\sum_{j=1}^K \|S_{kj}(t)\|^2 (N_0 + N_j(t))} \right), \quad (5)$$

where $N(t)$ is a function of $\mathbf{H}(t)$ and $\mathbf{C}(t)$, and $S(t)$ is a function of $\mathbf{H}(t)$. Note that there is an implicit coupling among the K uplink data flows in the sense that R_k depends not only on the fronthaul capacity allocation C_k but also on $C_j, \forall j \neq k$.

E. Queue Dynamics

There is a bursty data source for each UE. Let $\mathbf{A}(t) = (A_1(t)\tau, \dots, A_N(t)\tau)$ be the random arrivals (number of bits) from the application layers at the end of the t -th time slot³. We have the following assumption on $\mathbf{A}(t)$.

Assumption 3 (Bursty Source Model): Assume that $A_k(t)$ is i.i.d. over slots according to a general distribution $\Pr[A_k]$. The moment generating functions of A_k exist with $\mathbb{E}[A_k] = \lambda_k$. $A_k(t)$ is independent w.r.t. k . Furthermore, the arrival rates $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_K)$ lie within the stability region [14] of the system with the given uplink resource. ■

³We assume that the transmitters are causal so that the packets arrived at the time slot are not observed when the control actions of this time slot are performed.

Each UE has a data queue for the bursty traffic flows towards the associated RU. Let $Q_k(t) \in [0, \infty)$ be the queue length (number of bits) at the k -th UE at the beginning of the t -th slot. Let $\mathbf{Q}(t) = (Q_1(t), \dots, Q_N(t)) \in \mathcal{Q} \triangleq [0, \infty)^K$ be the *global QSI*. The queue dynamics for the k -th UE can be written as

$$Q_k(t+1) = \max \{Q_k(t) - R_k(\mathbf{H}(t), \mathbf{C}(t))\tau, 0\} + A_k(t)\tau. \quad (6)$$

Remark 1 (Coupling Property of Uplink Queue Dynamics): In the uplink, the K queue dynamics are coupled together due to the ZF processing in the BBU. Specifically, according to (5), the queue departure $R_k(\mathbf{H}(t), \mathbf{C}(t))$ for the i -th UE depends on not only the allocated capacity $C_k(t)$ for the k -th fronthaul link, but also all the other elements of $\mathbf{C}(t)$. ■

III. A CONTROL FRAMEWORK OF DELAY-AWARE UPLINK FRONTHAUL ALLOCATION

In this section, we formulate the delay-aware control framework of uplink fronthaul allocation. We first define the control policy and the optimization objective. We then formulate the design as a Markov Decision Process (MDP) and derive the optimality conditions for solving the problem.

A. Fronthaul Allocation Policy

For delay-sensitive applications, it is important to dynamically adapt the fronthaul capacities $\mathbf{C}(t)$ based on the instantaneous realizations of the CSI (captures the instantaneous transmission opportunities) and the QSI (captures the urgency of K data flows). Let $\chi = (\mathbf{H}, \mathbf{Q})$ denote the global system state. We define the stationary fronthaul allocation policy below:

Definition 1 (Stationary Fronthaul Allocation Policy): A stationary control policy for the k -th UE Ω_k is a mapping from the system state χ to the fronthaul allocation action of the k -th UE. Specifically, $\Omega_k(\chi) = C_k \geq 0$. Let $\mathbf{\Omega} = \{\Omega_k : \forall k\}$ denote the aggregation of the control policies for all the K UEs. ■

The CSI \mathbf{H} is i.i.d. over time slots based on the block fading channel model in Assumption 1. Furthermore, from the queue evolution equation in (6), $\mathbf{Q}(t+1)$ depends only on $\mathbf{Q}(t)$ and the data rate. Given a control policy $\mathbf{\Omega}$, the data rate at the t -th slot depends on $\mathbf{H}(t)$ and $\mathbf{\Omega}(\chi(t))$.

Hence, the global system state $\chi(t)$ is a controlled Markov chain [8] with the following transition probability:

$$\Pr[\chi(t+1)|\chi(t), \Omega(\chi(t))] = \Pr[\mathbf{H}(t+1)] \Pr[\mathbf{Q}(t+1)|\chi(t), \Omega(\chi(t))] \quad , \quad (7)$$

where the queue transition probability is given by

$$\Pr[\mathbf{Q}(t+1)|\chi(t), \Omega(\chi(t))] = \begin{cases} \prod_k \Pr[A_k(t)] & \text{if } Q_k(t+1) \text{ is given by (6), } \forall k \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

where the equality is due to the i.i.d. assumption of $\mathbf{H}(t)$ in Assumption 1.

For technical reasons, we consider the *admissible control policy* defined below.

Definition 2 (Admissible Control Policy): A policy Ω is admissible if the following requirements are satisfied:

- Ω is a unichain policy, i.e., the controlled Markov chain $\chi(t)$ under Ω has a single recurrent class (and possibly some transient states) [8].
- The queueing system under Ω is second-order stable in the sense that $\lim_{t \rightarrow \infty} \mathbb{E}^\Omega[\sum_{k=1}^K Q_k^2(t)] < \infty$, where \mathbb{E}^Ω means taking expectation w.r.t. the probability measure induced by the control policy Ω . ■

B. Problem Formulation

As a result, under an admissible control policy Ω , the average delay for the k -th data queue is given by

$$\overline{D}_k(\Omega) = \limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}^\Omega \left[\frac{Q_k(t)}{\lambda_k} \right], \quad \forall k. \quad (9)$$

Similarly, under an admissible control policy Ω , the average fronthaul capacity for the k -th data queue is given by

$$\overline{C}_k(\Omega) = \limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}^\Omega [C_k(t)], \quad \forall k. \quad (10)$$

We formulate the delay-aware fronthaul allocation problem for C-RANs as follows:

Problem 1 (Delay-Aware Fronthaul Allocation Problem): The delay-aware fronthaul allocation problem is formulated as

$$\begin{aligned} \min_{\Omega} \quad L(\Omega) &= \sum_{k=1}^K \left(\beta_k \overline{D}_k(\Omega) + \gamma_k \overline{C}_k(\Omega) \right) \\ &= \limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}^{\Omega} [c(\mathbf{Q}(t), \Omega(\chi(t)))] \end{aligned} \quad (11)$$

where $c(\mathbf{Q}, \mathbf{C}) = \sum_{k=1}^K \left(\beta_k \frac{Q_k}{\lambda_k} + \gamma_k C_k \right)$. $\beta = \{\beta_k > 0 : \forall k\}$ are the positive weights for the delay cost and $\gamma = \{\gamma_k > 0 : \forall k\}$ are the prices for the data transmission over fronthaul links.

■

Note that Problem 1 is an infinite horizon average cost MDP, which is known as a very difficult problem.

C. Optimality Conditions for Uplink Fronthaul Allocation

Problem 1 is an MDP and the *Bellman equation* [8] provides its optimality conditions. The Bellman equation involves the entire system state $\chi = (\mathbf{H}, \mathbf{Q})$. Exploiting the i.i.d. property of $\mathbf{H}(t)$ according to Assumption 1, we obtain the *equivalent Bellman equation* in the following theorem.

Theorem 1 (Sufficient Conditions for Optimality): For any given weights β , assume there exists a $(\theta^*, \{V^*(\mathbf{Q})\})$ that solves the following *equivalent Bellman equation*:

$$\theta^* \tau + V^*(\mathbf{Q}) = \mathbb{E} \left[\min_{\Omega(\chi)} \left[c(\mathbf{Q}, \Omega(\chi)) \tau + \sum_{\mathbf{Q}'} \Pr[\mathbf{Q}' | \chi, \Omega(\chi)] V^*(\mathbf{Q}') \right] \middle| \mathbf{Q} \right], \quad \forall \mathbf{Q} \in \mathcal{Q}, \quad (12)$$

Furthermore, for all admissible control policies Ω , V^* satisfies the following *transversality condition*:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}^{\Omega} [V^*(\mathbf{Q}(T))] = 0. \quad (13)$$

Then θ^* is the optimal average cost, and $V^*(\mathbf{Q})$ is the *priority function* of the K data flows. If there exists an admissible stationary policy $\Omega^*(\chi) = \mathbf{C}^*$ where \mathbf{C}^* attains the minimum of the R.H.S. of (12) for all $\mathbf{Q} \in \mathcal{Q}$, then Ω^* is the optimal control policy for Problem 1.

Proof: Please refer to Appendix A. ■

Remark 2 (Interpretation of Theorem 1): The equivalent Bellman equation in (12) is defined on the QSI \mathbf{Q} only. Nevertheless, the optimal control policy Ω^* obtained by solving (12) is still adaptive to the entire system state χ . At each stage, when the queue length is $\mathbf{Q}(t)$, the optimal action has to strike a balance between the current cost $c(\mathbf{Q}, \Omega(\chi))$ and the future cost $\sum_{\mathbf{Q}'} \Pr[\mathbf{Q}' | \chi, \Omega(\chi)] V^*(\mathbf{Q}')$ because the action taken will affect the future evolution of $\mathbf{Q}(t+1)$. ■

IV. LOW-COMPLEXITY FRONTHAUL ALLOCATION

One key obstacle in deriving the optimal fronthaul policy Ω^* is to obtain the priority function $V^*(\mathbf{Q})$ of the Bellman equation in (12). Conventional brute force value iteration or policy iteration algorithms can only give numerical solutions and have exponential complexity in K , which is highly undesirable. In this section, we shall exploit the characteristics of the topology of C-RANs. Specifically, we define $\delta = \max\{L_{kj} : \forall k \neq j\}$ be the worst-case path gain among all the cross links, which is usually weaker than the home cell path gain due to the C-RAN network architecture. We adopt perturbation theory w.r.t. δ to obtain a closed-form approximation of the priority function $V^*(\mathbf{Q})$ and derive the associated error bound. Based on that, we obtain a low complexity delay-aware fronthaul allocation algorithm.

A. Calculus Approach for Solving the Bellman Equation

We adopt a calculus approach to obtain a closed-form approximate priority function. We first have the following theorem for solving the Bellman equation in (12).

Theorem 2 (Calculus Approach for Solving (12)): Assume there exist c^∞ and $J(\mathbf{Q}; \delta)$ of class $\mathcal{C}^2(\mathbb{R}_+^K)$ that satisfy

- the following partial differential equation (PDE):

$$\mathbb{E} \left[\min_{\Omega(\chi)} \left[\sum_{k=1}^K \left(\beta_k \frac{Q_k}{\lambda_k} + \gamma_k C_k \right) - c^\infty + \sum_{k=1}^K \left(\frac{\partial J(\mathbf{Q}; \delta)}{\partial Q_k} (\lambda_k - R_k(\mathbf{H}, \mathbf{C})) \right) \right] \middle| \mathbf{Q} \right] = 0, \quad \forall \mathbf{Q} \in \mathbb{R}_+^K \quad (14)$$

with boundary condition $J(\mathbf{0}; \delta) = 0$;

- For all k , $\frac{\partial J(\mathbf{Q}; \delta)}{\partial Q_k}$ is an increasing function of all Q_k ;
- $J(\mathbf{Q}; \delta) = \mathcal{O}(\|\mathbf{Q}\|^2)$.

Then, we have

$$\theta^* = c^\infty + o(1), V^*(\mathbf{Q}) = J(\mathbf{Q}; \delta) + o(1), \forall \mathbf{Q} \in \mathcal{Q}, \quad (15)$$

where the error term $o(1)$ asymptotically goes to zero for sufficiently small τ .

Proof: Please refer to Appendix B. ■

Theorem 2 suggests that if we can solve for the PDE in (14), then the solution $(J(\mathbf{Q}; \delta), c^\infty)$ is only $o(1)$ away from the solution of the Bellman equation $(V^*(\mathbf{Q}), \theta^*)$.

B. Closed-Form Approximate Priority Function via Perturbation Analysis

The queues of the K uplink data flows are coupled due to the coupling of R_k in (5). The following lemma establishes the intensity of the queue coupling.

Lemma 1 (Intensity of the Uplink Queue Coupling): The coupling intensity of uplink data queues induced by R_k in (5) is given by $\|S_{kj}(t)\|^2 = \mathcal{O}(\delta), \forall k \neq j$.

Proof: Please refer to Appendix C. ■

As a result, the solution of (14) depends on the worst-case cross link interference path gain δ and, hence, the K -dimensional PDE in (14) can be regarded as a perturbation of a *base system*, as defined below.

Definition 3 (Base System): A base system is characterized by the PDE in (14) with $\delta = 0$. ■

According to Lemma 1, we have $\|S_{kj}(t)\|^2 = 0, \forall k \neq j$ in the base system. We first study the base system and use $J(\mathbf{Q}; 0)$ to obtain a closed-form approximation of $J(\mathbf{Q}; \delta)$.

We have the following lemma summarizing the priority function $J(\mathbf{Q}; 0)$ of the base system.

Lemma 2 (Decomposable Structure of $J(\mathbf{Q}; 0)$): The solution $J(\mathbf{Q}; 0)$ for the base system has the following decomposable structure:

$$J(\mathbf{Q}; 0) = \sum_{k=1}^K J_k(Q_k), \quad (16)$$

where $J_k(Q_k)$ is the *per-flow priority function* for the k -th data flow given by

$$\begin{cases} Q_k(\nu) = \frac{\lambda_k}{\beta_k} \left(\frac{\nu e^{a_k}}{\ln 2} E_1 \left(\frac{a_k \nu}{\nu - \gamma_k} \right) - \lambda_k \nu - \frac{\gamma_k}{\ln 2} E_1 \left(\frac{a_k \gamma_k}{\nu - \gamma_k} \right) + c_k^\infty \right) \\ J_k(\nu) = \frac{\lambda_k}{2\beta_k \ln 2} \left(\gamma_k (\gamma_k - \nu) e^{\frac{a_k \gamma_k}{\gamma_k - \nu}} + e^{a_k} \nu^2 E_1 \left(\frac{a_k \nu}{\nu - \gamma_k} \right) \right. \\ \left. + (a_k - \lambda_k) E_1 \left(\frac{a_k \gamma_k}{\nu - \gamma_k} \right) - \lambda_k \nu^2 \ln 2 \right) + b_k, \end{cases} \quad (17)$$

where $a_k \triangleq \frac{N_0}{PL_{kk}}$; $c_k^\infty = \frac{\gamma_k}{\ln 2} E_1 \left(\frac{a_k \gamma_k}{d_k - \gamma_k} \right)$, where d_k satisfies $\frac{e^{a_k}}{\ln 2} E_1 \left(\frac{a_k d_k}{d_k - \gamma_k} \right) = \lambda_k$; $E_1(z) \triangleq \int_1^\infty \frac{e^{-tz}}{t} dt = \int_z^\infty \frac{e^{-t}}{t} dt$; b_k is chosen to satisfy⁴ the boundary condition $J_k(0) = 0$.

Proof: Please refer to Appendix D. ■

Note that when $\delta = 0$, we have $L_{kj} = 0$ for all $k \neq j$ and, hence, there is no coupling between the UE-RU pairs. As a result, the K data queues are totally decoupled and the system is equivalent to a decoupled system with K independent queues. That is why the priority function $J(\mathbf{Q}; 0)$ in the base system has the decomposable structure in Lemma 2.

When $\delta > 0$, $J(\mathbf{Q}; \delta)$ can be considered as a perturbation of the solution of the base system $J(\mathbf{Q}; 0)$. Using perturbation analysis on the PDE (14), we establish the following theorem on the approximation of $J(\mathbf{Q}; \delta)$:

Theorem 3 (First Order Perturbation of $J(\mathbf{Q}; \delta)$): $J(\mathbf{Q}; \delta)$ is given by

$$J(\mathbf{Q}; \delta) = J(\mathbf{Q}; 0) + \sum_{k=1}^K \left(\sum_{j=1, j \neq k}^K L_{kj} \left(\Phi_k Q_k^2 \sum_{l=1, l \neq k}^K \frac{N_0}{L_{ll}} + \frac{N_0}{L_{jj}} \sum_{i=1, i \neq k, j}^K \Phi_i Q_i^2 \right) + o(Q_k^2) \right) + \mathcal{O}\left(\frac{1}{\delta^2}\right), \quad (18)$$

where $\Phi_k = \frac{\beta_k}{\lambda_k} \left(1 - \frac{a_k e^{a_k} E_1(a_k)}{N_0} \right) / (e^{a_k} E_1(a_k) - \lambda_k \ln 2)$.

Proof: Please refer to Appendix E. ■

The priority function $V(\mathbf{Q})$ is decomposed into the following three terms: 1) the base term $\sum_k J_k(Q_k)$ obtained by solving a base system without coupling, 2) the perturbation term accounting for the first order coupling due to the joint processing in the BBU, and 3) the residual error term which goes to zero in the order of $\mathcal{O}(1/\delta^2)$. As a result, we adopt the

⁴To find b_k , firstly solve $Q_k(\nu) = 0$ using one-dimensional search techniques (e.g., bisection method). Then b_k is chosen such that $J_k(\nu) = 0$.

following closed-form approximation of $V(\mathbf{Q})$:

$$\tilde{V}(\mathbf{Q}) = \sum_{k=1}^K J_k(Q_k) + \sum_{k=1}^K \left(\sum_{j=1, j \neq k}^K L_{kj} \left(\Phi_k Q_k^2 \sum_{l=1, l \neq k}^K \frac{N_0}{L_{ll}} + \frac{N_0}{L_{jj}} \sum_{i=1, i \neq k, j}^K \Phi_i Q_i^2 \right) \right). \quad (19)$$

C. Fronthaul Allocation Algorithm

In this section, we use the closed-form approximate priority function in (19) to capture the urgency information of the K data flows and obtain a low complexity delay-aware fronthaul allocation algorithm. Using the approximate priority function in (19), the per-stage control problem (for each state realization χ) is given by⁵

$$\max_{\mathbf{C}} \sum_{k=1}^K \left(\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} R_k(\mathbf{H}, \mathbf{C}) - \gamma_k C_k \right), \quad (20)$$

where $\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k}$ can be calculated from (19), which is given by

$$\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} = J'_k(Q_k) + 2\Phi_k \left(\sum_{j=1, j \neq k}^K L_{kj} \sum_{l=1, l \neq k}^K \frac{N_0}{L_{ll}} + \sum_{i=1, i \neq k}^K \sum_{j=1, j \neq i, k}^K L_{ij} \frac{N_0}{L_{jj}} \right) Q_k. \quad (21)$$

The per-stage problem in (20) is similar to the weighted sum-rate (WSR) optimization [15], which can be considered as a special case of network utility maximization. However, unlike conventional WSR problems, where the weights are static, the weights here in (20) are dynamic and are determined by the QSI via the priority function $\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k}$. As such, the role of the QSI is to dynamically adjust the weight (priority) of the individual flows, whereas the role of the CSI is to adjust the priority of the flow based on the transmission opportunity in the rate function $R_k(\mathbf{H}, \mathbf{C})$.

One approach to solve the WSR problem is solving the local optimization problem for each flow iteratively [15]. In each local optimization problem for the k -th flow, the total WSR objective is maximized, assuming that the capacities of other links $C_j, \forall j \neq k$ do not change. The local optimization problem is formulated as

$$\max_{C_k} \sum_{k=1}^K \left(\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} R_k(\mathbf{H}, \mathbf{C}) - \gamma_k C_k \right). \quad (22)$$

⁵Note that $J'_k(Q_k) = \left(\frac{dJ_k(\nu)}{d\nu} \right) \bigg|_{\nu=\nu(Q_k)} = \nu(Q_k)$, where $\nu(Q_k)$ satisfies $Q_k(\nu(Q_k)) = Q_k$.

The above local optimization problem is still difficult to solve directly. An alternative method is simplifying the effect of C_k on the other links as a linear function⁶ [16]. Define π_{ik} as the marginal increase in the utility of the i -th flow per unit increase in C_k , i.e.,

$$\pi_{ik} = \frac{\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_i} P \|S_{ik}\|^2 Y_k \frac{2^{C_k}}{(2^{C_k}-1)^2}}{\left(P + I_{ik} + \|S_{ik}\|^2 \left(N_0 + \frac{Y_k}{2^{C_k-1}}\right)\right) \left(I_{ik} + \|S_{ik}\|^2 \left(N_0 + \frac{Y_k}{2^{C_k-1}}\right)\right)}, \quad (23)$$

where $I_{ik} = \sum_{j=1, j \neq k}^K \|S_{ij}\|^2 \left(N_0 + \frac{Y_j}{2^{C_j-1}}\right)$ and $Y_j = P \sum_{l=1}^K \|H_{jl}\|^2 + N_0$.

Adopting the linear simplification $\pi_{ik} C_k$ for the effect of C_k on the i -th flow in the per-stage local optimization problem (22), we have the Karush-Kuhn-Tucker (KKT) condition as

$$\frac{\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} P \|S_{kk}\|^2 Y_k \frac{2^{C_k}}{(2^{C_k}-1)^2}}{\left(I_{kk} + \|S_{kk}\|^2 \left(N_0 + \frac{Y_k}{2^{C_k-1}}\right)\right) \left(P + I_{kk} + \|S_{kk}\|^2 \left(N_0 + \frac{Y_k}{2^{C_k-1}}\right)\right)} + \sum_{i=1, i \neq k}^K \pi_{ik} = \gamma. \quad (24)$$

By solving (24), we obtain the optimal fronthaul capacity C_k for the local optimization problem as

$$C_k = \log_2 \frac{\eta_k + \zeta_k + \sqrt{\eta_k^2 + 2\eta_k \zeta_k + P^2 \|S_{kk}\|^4 Y_k^2}}{2 \left(P + I_{kk} + \|S_{kk}\|^2 N_0\right) \left(I_{kk} + \|S_{kk}\|^2 N_0\right)}, \quad (25)$$

where $\zeta_k = 2I_{kk}^2 + 2I_{kk}(P + 2\|S_{kk}\|^2 N_0 - \|S_{kk}\|^2 Y_k) + \|S_{kk}\|^2 (2\|S_{kk}\|^2 N_0^2 - PY_k + 2PN_0 - 2\|S_{kk}\|^2 N_0 Y_k)$ and $\eta_k = \frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} \frac{P \|S_{kk}\|^2 Y_k}{\gamma - \sum_{i=1, i \neq k}^K \pi_{ik}}$.

Based on the above analysis, we propose a low-complexity fronthaul allocation algorithm launched at the beginning of each slot, which is described using pseudo codes as Algorithm 1. We denote $\mathbf{C}^{(n)} = (C_1^{(n)}, C_2^{(n)}, \dots, C_K^{(n)})$ as the allocated fronthaul capacities in the n -th iteration.

Although the per-stage problem (20) is not convex in general, the following lemma states that it is a convex problem for sufficiently small δ .

Lemma 3 (Asymptotic Convexity): When δ is sufficiently small, the objective in (20) is a concave function of \mathbf{C} , and the problem (20) is a convex problem.

Proof: Please refer to Appendix F. ■

⁶We will show later that this simplification does not affect the convergence property.

Algorithm 1 Delay-Aware Fronthaul Allocation

- 1: Initialize $n = 0$ and $C_k^{(0)} = 0, \forall k$
 - 2: **repeat**
 - 3: **for** all user k **do**
 - 4: Calculate $C_k^{(n+1)}$ based on $\mathbf{C}^{(n)}$ according to (25)
 - 5: **end for**
 - 6: $n = n + 1$
 - 7: **until** The difference between $\mathbf{C}^{(n)}$ and $\mathbf{C}^{(n+1)}$ is below a given threshold
-

According to Lemma 3, we provide the convergence property and asymptotic optimality of Algorithm 1 in the following theorem:

Theorem 4 (Asymptotic Optimality): When δ is sufficiently small, starting from any feasible initial point $\mathbf{C}^{(0)}$, Algorithm 1 converges to the optimal solution of the original Problem 1.

Proof: Please refer to Appendix G. ■

V. SIMULATION

In this section, we evaluate the performance of the proposed low-complexity delay-aware fronthaul allocation algorithm for C-RANs. For performance comparison, we adopt the following two baseline schemes.

- **Baseline 1 [Throughput-Optimal Fronthaul Allocation]:** The throughput-optimal fronthaul allocation algorithm determines the fronthaul capacities for maximizing the total data rate without considering the queueing information, which is similar to that in [6] but with ZF processing.
- **Baseline 2 [Queue-Weighted Fronthaul Allocation]:** The queue-weighted fronthaul allocation algorithm exploits both CSI and QSI for queue stability by Lyapunov drift [14] and solves the per-stage problem (20) replacing $\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k}$ with Q_k [17].

In the simulation, the performance of the proposed fronthaul allocation algorithm is evaluated in a C-RAN cluster with seven cells. A single channel is considered, and one user over the

channel is located randomly in each cell, with radius 500m. Poisson data arrival is considered, with an average arrival rate λ_k for the k -th UE, which is uniformly distributed between $[0, 2\bar{\lambda}]$ with mean $\bar{\lambda}$. The path gain is calculated as $L_{kj} = 15.3 + 37.6 \log_{10} d_{kj}$, with the fading coefficient distributed as $\mathcal{CN}(0, 1)$. The average transmit power is 23dBm and the noise power spectrum density is -174dBm/Hz. The system bandwidth is 10MHz and the duration of the decision slot is 10ms. The weights γ_k are the same and $\beta_k = 1$ for all k . For comparison, the delay performances of different schemes are evaluated with the same total fronthaul capacity by adjusting γ_k . For obtaining the average performance, we consider 20 random topologies, each of which has 100 time slots.

Fig. 3 shows the average delay versus the average arrival rate when the total fronthaul capacity is 350Mbps. For all algorithms, the average delay increases when the average traffic load increases. It can be observed that the proposed fronthaul allocation algorithm outperforms both baselines, which verifies the accuracy of the priority function approximation in the proposed algorithm.

Fig. 4 shows the average delay versus the total fronthaul capacity when the average arrival rate is 30Mbps. The proposed fronthaul allocation algorithm also achieves better performance than the baseline schemes. When the total fronthaul capacity is small, the average delay decreases significantly with the increase of the total fronthaul capacity. In contrast, when the total fronthaul capacity is large, the change in the average delay is relatively small with adjustment of the total fronthaul capacity.

Table I illustrates a comparison of the MATLAB computational time of the proposed solution, the baselines and the brute-force value iteration algorithm [8] in one time slot. From the results, our proposed algorithm has much less complexity than the brute-force value iteration algorithm. The computational time of our proposed algorithm is close to those of Baselines 1 & 2, and the difference is due to the computation of the approximate priority function. Therefore, our proposed algorithm achieves significant performance gain compared to the baselines, with small computational complexity cost.

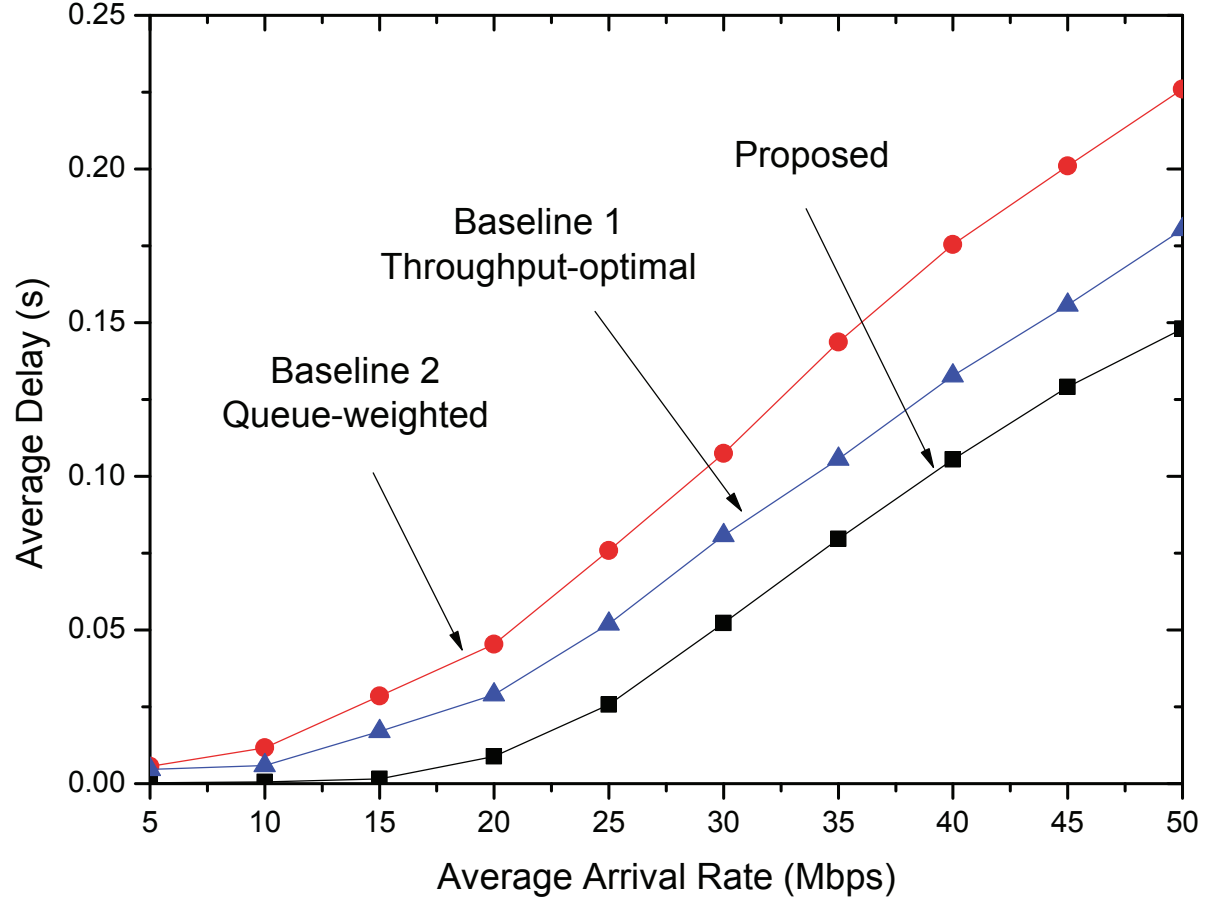


Figure 3. Performance comparison with different average arrival rates when $\bar{C} = 350\text{Mbps}$

Table I
COMPARISON OF THE MATLAB COMPUTATIONAL TIME

Algorithm	Time
Baselines 1 & 2	0.006s
Proposed Algorithm	0.043s
Brute-Force Value Iteration	$> 10^5\text{s}$

VI. CONCLUSIONS

In this paper, we propose a low-complexity delay-aware fronthaul allocation algorithm for the uplink in C-RANs. The delay-aware fronthaul allocation problem is formulated as an infinite

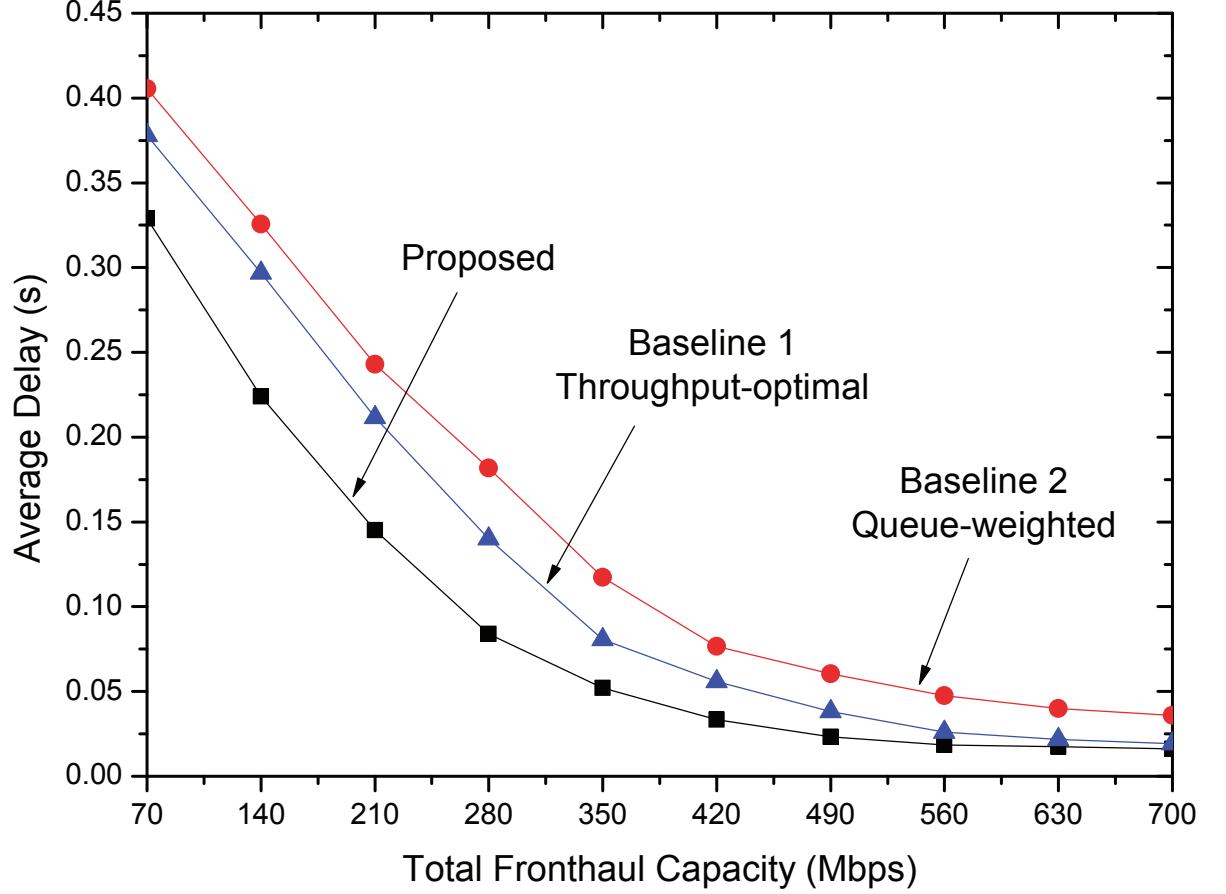


Figure 4. Performance comparison with different total fronthaul capacity when $\bar{\lambda} = 30\text{Mbps}$

horizon average cost Markov decision process. To deal with the curse of dimensionality, we exploit the specific problem structure that the cross link path gain is usually weaker than the home cell path gain. Utilizing the perturbation analysis technique, we obtain a closed-form approximate priority function and the associated error bound. Based on the closed-form approximate priority function, we propose a low-complexity delay-aware fronthaul allocation algorithm, solving the per-stage optimization problem. The proposed solution is further shown to be asymptotically optimal for sufficiently small cross link path gains. The simulation results verify the accuracy of the priority function approximation and show that the proposed fronthaul allocation algorithm outperforms the baselines.

APPENDIX A

PROOF OF THEOREM 1

Following *Prop. 4.6.1* of [8], the sufficient conditions for the optimality of *Problem 1* are that $(\theta^*, \{V^*(\mathbf{Q})\})$ solves the following Bellman equation:

$$\begin{aligned} \theta^* \tau + V^*(\chi) &= \min_{\Omega(\chi)} \left[c(\mathbf{Q}, \Omega(\chi)) \tau + \sum_{\chi'} \Pr[\chi' | \chi, \Omega(\chi)] V^*(\chi') \right] \\ &= \min_{\Omega(\chi)} \left[c(\mathbf{Q}, \Omega(\chi)) \tau + \sum_{\mathbf{Q}'} \sum_{\mathbf{H}'} \Pr[\mathbf{Q}' | \chi, \Omega(\chi)] \Pr[\mathbf{H}'] V^*(\chi') \right] \end{aligned} \quad (26)$$

and V^* satisfies the condition in (13) for all admissible policies Ω . Then $\theta^* = \min_{\Omega(\chi)} L(\Omega(\chi))$.

Taking expectation w.r.t. \mathbf{H} on both sides of (26) and denoting $V^*(\mathbf{Q}) = \mathbb{E}[V^*(\chi) | \mathbf{Q}]$, we obtain the equivalent Bellman equation in (12) in Theorem 1.

APPENDIX B

PROOF OF THEOREM 2

In the proof, we shall first establish the relationship between the equivalent Bellman equation in (12) in Theorem 2 and the approximate Bellman equation in (27) in the following Lemma 4. Then, we establish the relationship between the approximate Bellman equation in (27) in Lemma 4 and the PDE in (14) in Theorem 2.

1. Relationship between the Equivalent Bellman and Approximate Bellman Equations

We establish the following lemma on the approximate Bellman equation to simplify the equivalent Bellman equation in (12):

Lemma 4 (Approximate Bellman Equation): For any given weights β , if

- there is a unique $(\theta^*, \{V^*(\mathbf{Q})\})$ that satisfies the Bellman equation and transversality condition in Theorem 1;
- there exist θ and $V(\mathbf{Q})$ of class⁷ $\mathcal{C}^2(\mathbb{R}_+^K)$ that solve the following *approximate Bellman*

⁷ $f(\mathbf{x})$ (\mathbf{x} is a K -dimensional vector) is of class $\mathcal{C}^2(\mathbb{R}_+^K)$ if the first and second order partial derivatives of $f(\mathbf{x})$ w.r.t. each element of \mathbf{x} are continuous when $\mathbf{x} \in \mathbb{R}_+^K$.

equation:

$$\theta = \mathbb{E} \left[\min_{\Omega(\chi)} \left[c(\mathbf{Q}, \Omega(\chi)) + \sum_{k=1}^K \frac{\partial V(\mathbf{Q})}{\partial Q_k} \left[\lambda_k - R_k(\mathbf{H}, \Omega(\chi)) \right] \right] \middle| \mathbf{Q} \right], \forall \mathbf{Q} \in \mathcal{Q} \quad (27)$$

and for all admissible control policies Ω , the transversality condition in (13) is satisfied for V ,

then, we have

$$\theta^* = \theta + o(1), \quad V^*(\mathbf{Q}) = V(\mathbf{Q}) + o(1), \quad \forall \mathbf{Q} \in \mathcal{Q}, \quad (28)$$

where the error term $o(1)$ asymptotically goes to zero for sufficiently small slot duration τ . ■

Proof of Lemma 4: Let $\mathbf{Q}' = (Q'_1, \dots, Q'_k) = \mathbf{Q}(t+1)$ and $\mathbf{Q} = (Q_1, \dots, Q_k) = \mathbf{Q}(t)$. For the queue dynamics in (6) and sufficiently small τ , we have $Q'_k = Q_k - R_k(\mathbf{H}, \mathbf{C})\tau + A_k\tau$, ($\forall k$). Therefore, if $V(\mathbf{Q})$ is of class $\mathcal{C}^2(\mathbb{R}_+^K)$, we have the following Taylor expansion on $V(\mathbf{Q}')$:

$$\mathbb{E}[V(\mathbf{Q}') | \mathbf{Q}] = V(\mathbf{Q}) + \sum_{k=1}^K \frac{\partial V(\mathbf{Q})}{\partial Q_k} \left(\lambda_k - \mathbb{E}[R_k(\mathbf{H}, \Omega(\chi)) | \mathbf{Q}] \right) \tau + o(\tau). \quad (29)$$

For notation convenience, let $F_{\chi}(\theta, V, \Omega(\chi))$ denote the *Bellman operator*:

$$F_{\chi}(\theta, V, \Omega(\chi)) = \sum_{k=1}^K \frac{\partial V(\mathbf{Q})}{\partial Q_k} (\lambda_k - R_k(\mathbf{H}, \Omega(\chi))) - \theta + c(\mathbf{Q}, \Omega(\chi)) + \nu G_{\chi}(V, \Omega(\chi)) \quad (30)$$

for some smooth function G_{χ} and $\nu = o(1)$ (w.r.t. τ). Denote $F_{\chi}(\theta, V) = \min_{\Omega(\mathbf{Q})} F_{\chi}(\theta, V, \Omega(\chi))$. Suppose (θ^*, V^*) satisfies the Bellman equation in (12), we have $\mathbb{E}[F_{\chi}(\theta^*, V^*) | \mathbf{Q}] = 0$, $\forall \mathbf{Q} \in \mathcal{Q}$. Similarly, if (θ, V) satisfies the approximate Bellman equation in (27), we have

$$\mathbb{E}[F_{\chi}^{\dagger}(\theta, V) | \mathbf{Q}] = 0, \quad \forall \mathbf{Q} \in \mathcal{Q}, \quad (31)$$

where $F_{\chi}^{\dagger}(\theta, V) = \min_{\Omega(\mathbf{Q})} F_{\chi}^{\dagger}(\theta, V, \Omega(\chi))$ and $F_{\chi}^{\dagger}(\theta, V, \Omega(\chi)) = F_{\chi}(\theta, V, \Omega(\chi)) - \nu G_{\chi}(V, \Omega(\chi))$. We then establish the following lemma.

Lemma 5: If (θ, V) satisfies the approximate Bellman equation in (27), then $|\mathbb{E}[F_{\chi}(\theta, V) | \mathbf{Q}]| = o(1)$ for any $\mathbf{Q} \in \mathcal{Q}$. ■

Proof of Lemma 5: For any χ , we have $F_{\chi}(\theta, V) = \min_{\Omega(\chi)} [F_{\chi}^{\dagger}(\theta, V, \Omega(\chi)) + \nu G_{\chi}(V, \Omega(\chi))] \geq \min_{\Omega(\chi)} F_{\chi}^{\dagger}(\theta, V, \Omega(\chi)) + \nu \min_{\Omega(\chi)} G_{\chi}(V, \Omega(\chi))$. Besides this, $F_{\chi}(\theta, V) \leq$

$\min_{\Omega(\chi)} F_{\chi}^{\dagger}(\theta, V, \Omega(\chi)) + \nu G_{\chi}(V, \Omega^{\dagger}(\chi))$, where $\Omega^{\dagger} = \arg \min_{\Omega(\chi)} F_{\chi}^{\dagger}(\theta, V, \Omega(\chi))$. Since $\mathbb{E}[\min_{\Omega(\chi)} F_{\chi}^{\dagger}(\theta, V, \Omega(\chi)) | \mathbf{Q}] = 0$ according to (31), and F_{χ}^{\dagger} and G_{χ} are all smooth and bounded functions, we have $|\mathbb{E}[F_{\chi}(\theta, V) | \mathbf{Q}]| = o(1)$ (w.r.t. τ). ■

We establish the following lemma to prove Lemma 4.

Lemma 6: Suppose $\mathbb{E}[F_{\chi}(\theta^*, V^*) | \mathbf{Q}] = 0$ for all \mathbf{Q} together with the transversality condition in (13) has a unique solution (θ^*, V^*) . If (θ, V) satisfies the approximate Bellman equation in (27) and the transversality condition in (13), then $\theta = \theta^* + o(1)$, $V(\mathbf{Q}) = V^*(\mathbf{Q}) + o(1)$ for all \mathbf{Q} , where $o(1)$ asymptotically goes to zero as τ goes to zero. ■

Proof of Lemma 6: Suppose for some \mathbf{Q}' , $V(\mathbf{Q}') = V^*(\mathbf{Q}') + \mathcal{O}(1)$ (w.r.t. τ). From Lemma 5, we have $|\mathbb{E}[F_{\chi}(\theta, V) | \mathbf{Q}]| = o(1)$ (w.r.t. τ). Letting $\tau \rightarrow 0$, we have $\mathbb{E}[F_{\chi}(\theta, V) | \mathbf{Q}] = 0$ for all \mathbf{Q} and the transversality condition in (13). However, $V(\mathbf{Q}') \neq V^*(\mathbf{Q}')$ due to $V(\mathbf{Q}') = V^*(\mathbf{Q}') + \mathcal{O}(1)$. This contradicts the condition that (θ^*, V^*) is a unique solution of $F_{\chi}(\theta^*, V^*) = 0$ for all \mathbf{Q} and the transversality condition in (13). Hence, we must have $V(\mathbf{Q}) = V^*(\mathbf{Q}) + o(1)$ for all \mathbf{Q} . Similarly, we can establish $\theta = \theta^* + o(1)$. ■

2. Relationship between the Approximate Bellman Equation and the PDE

For notation convenience, we write $J(\mathbf{Q})$ in place of $J(\mathbf{Q}; \delta)$. It can be observed that if $(c^{\infty}, \{J(\mathbf{Q})\})$ satisfies (14), it also satisfies (27). Furthermore, since $J(\mathbf{Q}) = \mathcal{O}(\sum_{k=1}^K Q_k^2)$, then $\lim_{t \rightarrow \infty} \mathbb{E}^{\Omega} [J(\mathbf{Q}(t))] < \infty$ for any admissible policy Ω . Hence, $J(\mathbf{Q}) = \mathcal{O}(\sum_{k=1}^K Q_k^2)$ satisfies the transversality condition in (13). Next, we show that the optimal policy Ω^{J^*} obtained from (14) is an admissible control policy according to Definition 2.

Define a *Lyapunov function* as $L(\mathbf{Q}) = J(\mathbf{Q})$. We define the *conditional queue drift* as $\Delta(\mathbf{Q}) = \mathbb{E}^{\Omega^{J^*}} [\sum_{k=1}^K (Q_k(t+1) - Q_k(t)) | \mathbf{Q}(t) = \mathbf{Q}]$ and the *conditional Lyapunov drift* as $\Delta L(\mathbf{Q}) = \mathbb{E}^{\Omega^{J^*}} [L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t) = \mathbf{Q}]$. We first have the following relationship between $\Delta(\mathbf{Q})$ and $\Delta L(\mathbf{Q})$:

$$\Delta L(\mathbf{Q}) \geq \mathbb{E}^{\Omega^{J^*}} \left[\sum_{k=1}^K \frac{\partial L(\mathbf{Q})}{\partial Q_k} (Q_k(t+1) - Q_k(t)) \middle| \mathbf{Q}(t) = \mathbf{Q} \right] \stackrel{(a)}{\geq} \Delta(\mathbf{Q}) \quad (32)$$

if at least one of $\{Q_k : \forall k\}$ is sufficiently large, where (a) is due to the condition that $\frac{\partial J(\mathbf{Q})}{\partial Q_k}$ is

an increasing function of all Q_k .

Since $(\lambda_1, \dots, \lambda_K)$ is strictly interior to the stability region Λ , there exists $\bar{\lambda} = (\lambda_1 + \kappa_1, \dots, \lambda_K + \kappa_K) \in \Lambda$ for some positive $\kappa = \{\kappa_k : \forall k\}$ [14]. From *Corollary 1* of [18], there exists a stationary randomized QSI-independent policy $\tilde{\Omega}$ such that

$$\sum_{k=1}^K \mathbb{E}^{\tilde{\Omega}} [\gamma_k C_k | \mathbf{Q}(t) = \mathbf{Q}] = \tilde{C}(\kappa) \quad (33)$$

$$\mathbb{E}^{\tilde{\Omega}} [R_k(\mathbf{H}, \mathbf{C}) | \mathbf{Q}(t) = \mathbf{Q}] \geq \lambda_k + \kappa_k, \quad \forall k, \quad (34)$$

where $\tilde{C}(\kappa)$ is the minimum time-averaging total fronthaul capacity for the system stability when the arrival rate is $\bar{\lambda}$. The Lyapunov drift $\Delta L(\mathbf{Q})$ is given by

$$\begin{aligned} \Delta L(\mathbf{Q}) &+ \mathbb{E}^{\Omega^{J^*}} \left[\sum_{k=1}^K \gamma_k C_k \tau \middle| \mathbf{Q}(t) = \mathbf{Q} \right] \\ &\approx \sum_{k=1}^K \frac{\partial L(\mathbf{Q})}{\partial Q_k} \lambda_k \tau + \mathbb{E}^{\Omega^{J^*}} \left[\sum_{k=1}^K \left(\gamma_k C_k \tau - \frac{\partial L(\mathbf{Q})}{\partial Q_k} R_k(\mathbf{H}, \mathbf{C}) \tau \right) \middle| \mathbf{Q}(t) = \mathbf{Q} \right] \\ &\stackrel{(b)}{\leq} \sum_{k=1}^K \frac{\partial L(\mathbf{Q})}{\partial Q_k} \lambda_k \tau + \mathbb{E}^{\tilde{\Omega}} \left[\sum_{k=1}^K \left(\gamma_k C_k \tau - \frac{\partial L(\mathbf{Q})}{\partial Q_k} R_k(\mathbf{H}, \mathbf{C}) \tau \right) \middle| \mathbf{Q}(t) = \mathbf{Q} \right] \\ &\stackrel{(c)}{\leq} - \sum_{k=1}^K \frac{\partial L(\mathbf{Q})}{\partial Q_k} \kappa_k \tau + \tilde{C}(\kappa) \tau \end{aligned} \quad (35)$$

if at least one of $\{Q_k : \forall k\}$ is sufficiently large, where (b) is because Ω^{J^*} achieves the minimum of (14) and (c) is due to (33) and (34). Combining (35) with (32), we have $\Delta(\mathbf{Q}) \leq \Delta L(\mathbf{Q}) \leq - \sum_{k=1}^K \frac{\partial L(\mathbf{Q})}{\partial Q_k} \kappa_k \tau + \tilde{C}(\kappa) \tau < 0$ if at least one of $\{Q_k : \forall k\}$ is sufficiently large. Therefore, $\mathbb{E}[A_k - G_k(\mathbf{H}, \Omega^{J^*}(\chi)) | \mathbf{Q}] < 0$ when $Q_k > \bar{Q}_k$ for some large \bar{Q}_k . Let $\phi_k(r, \mathbf{Q}) = \ln(\mathbb{E}[e^{(A_k - G_k(\mathbf{H}, \Omega^{J^*}(\chi)))r} | \mathbf{Q}])$ be the *semi-invariant moment generating function* of $A_k - G_k(\mathbf{H}, \Omega^{J^*}(\chi))$. Then, $\phi_k(r, \mathbf{Q})$ will have a unique positive root $r_k^*(\mathbf{Q})$ ($\phi_k(r_k^*(\mathbf{Q}), \mathbf{Q}) = 0$) [19]. Let $r_k^* = r_k^*(\bar{\mathbf{Q}})$, where $\bar{\mathbf{Q}} = (\bar{Q}_1, \dots, \bar{Q}_K)$. Using the Kingman bound [19] result that

$F_k(x) \triangleq \Pr [Q_k \geq x] \leq e^{-r_k^* x}$, if $x \geq \bar{x}_k$ for sufficiently large \bar{x}_k , we have

$$\begin{aligned}
& \mathbb{E}^{\Omega^{J^*}} [J(\mathbf{Q})] \\
& \leq C \sum_{k=1}^K \mathbb{E}^{\Omega^{J^*}} [Q_k^2] = C \sum_{k=1}^K \left[\int_0^\infty \Pr [Q_k^2 > s] ds \right] \\
& \leq C \sum_{k=1}^K \left[\int_0^{\bar{x}_k^2} F_k(s^{1/2}) ds + \int_{\bar{x}_k^2}^\infty F_k(s^{1/2}) ds \right] \\
& \leq C \sum_{k=1}^K \left[\bar{x}_k^2 + \int_{\bar{x}_k^2}^\infty e^{-r_k^* s^{1/2}} ds \right] < \infty
\end{aligned} \tag{36}$$

for some constant C . Therefore, Ω^{J^*} is an admissible control policy and we have $V(\mathbf{Q}) = J(\mathbf{Q})$ and $\theta = c^\infty$.

Combining Lemma 4, we have $V^*(\mathbf{Q}) = J(\mathbf{Q}) + o(1)$ and $\theta^* = c^\infty + o(1)$ for sufficiently small τ .

APPENDIX C

PROOF OF LEMMA 1

The coupling among the K uplink data queues is induced by $\mathbf{S}(t)$ in the expression of R_k in (5). According to Assumption 2, $\mathbf{S}(t) = \mathbf{H}(t)^{-1}$. The time index t is omitted in this proof for simplicity of expression. We adopt the adjoint matrix to obtain the inverse of the channel matrix \mathbf{H} as

$$\mathbf{S} = \frac{1}{\det(\mathbf{H})} \text{adj}(\mathbf{H}) = \frac{1}{|\mathbf{H}|} \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1K} \\ M_{21} & M_{22} & \cdots & M_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ M_{K1} & M_{K2} & \cdots & M_{KK} \end{bmatrix}^T, \tag{37}$$

where M_{kj} is the (k, j) algebraic cofactor, which is the determinant of the submatrix formed by deleting the k -th row and j -th column of \mathbf{H} multiplied by $(-1)^{k+j}$.

With $\delta = \max \{L_{kj} : \forall k \neq j\}$, we can rewrite the channel matrix \mathbf{H} as

$$\begin{bmatrix} \mathcal{O}(1) & \mathcal{O}(\sqrt{\delta}) & \cdots & \mathcal{O}(\sqrt{\delta}) \\ \mathcal{O}(\sqrt{\delta}) & \mathcal{O}(1) & \cdots & \mathcal{O}(\sqrt{\delta}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{O}(\sqrt{\delta}) & \mathcal{O}(\sqrt{\delta}) & \cdots & \mathcal{O}(1) \end{bmatrix}, \quad (38)$$

where the K diagonal entries are $\mathcal{O}(1)$ and the other entries are $\mathcal{O}(\sqrt{\delta})$.

If $k \neq j$, the submatrix formed by deleting the k -th row and j -th column of \mathbf{H} includes $K - 2$ diagonal entries of \mathbf{H} , i.e., $\mathcal{O}(1)$. As a result, when calculating the determinant of the submatrix, each term of the determinant is the product of $K - 1$ entries and at least one $\mathcal{O}(\sqrt{\delta})$ term is included. Therefore, we obtain the coupling intensity $\|S_{kj}(t)\|^2 = \mathcal{O}(\delta)$, $\forall k \neq j$, and Lemma 1 holds.

APPENDIX D

PROOF OF LEMMA 2

We first prove that $J(\mathbf{Q}; 0) = \sum_{k=1}^K J_k(Q_k)$. The PDE in (14) for the base system is

$$\mathbb{E} \left[\min_{\Omega(\mathbf{x})} \left[\sum_{k=1}^K \left(\beta_k \frac{Q_k}{\lambda_k} + \gamma_k C_k + \frac{\partial J(\mathbf{Q}; 0)}{\partial Q_k} (\lambda_k - R_k(\mathbf{H}, \mathbf{C})) \right) \right] \middle| \mathbf{Q} \right] - c^\infty = 0. \quad (39)$$

We have the following lemma to prove the decomposable structures of $J(\mathbf{Q}; 0)$ and c^∞ in (39).

Lemma 7 (Decomposed Optimality Equation): Suppose there exist c_k^∞ and $J_k(Q_k) \in \mathbb{C}^2(\mathbb{R}_+)$ that solve the following per-flow optimality equation (PFOE):

$$\mathbb{E} \left[\min_{C_k \geq 0} \left[\beta_k \frac{Q_k}{\lambda_k} + \gamma_k C_k + J'_k(Q_k) (\lambda_k - R_k^0(H_{kk}, C_k)) \right] \middle| Q_k \right] - c_k^\infty = 0, \quad (40)$$

where $R_k^0(H_{kk}, C_k) = \log_2 \left(1 + \frac{P \|H_{kk}\|^2}{N_0 + N_k} \right)$ and $N_k = \frac{P \|H_{kk}\|^2 + N_0}{2^{C_k} - 1}$. Then, $J(\mathbf{Q}; 0) = \sum_{k=1}^K J_k(Q_k)$ and $c^\infty = \sum_{k=1}^K c_k^\infty$ satisfy (39). \blacksquare

Lemma 7 can be proved using the fact that the dynamics of the K queues at the UEs are decoupled when $\delta = 0$. The details are omitted for conciseness.

Next, we solve the optimization problem in (40). The optimal fronthaul capacity C_k^* from (40) is given by

$$C_k^* = \left(\log_2 \left(\frac{P \|H_{kk}\|^2}{N_0} \left(\frac{J'_k(Q_k)}{\gamma_k} - 1 \right)^+ \right) \right)^+. \quad (41)$$

Substituting the optimal allocated fronthaul capacity C_k^* into (40), and using the fact that $\|H_{kk}\|^2$ follows a negative exponential distribution with mean L_{kk} according to Assumption 1, we calculate the expectations in (40) as follows:

If $J'_k(Q_k) > \gamma_k$, the expected fronthaul capacity is

$$\begin{aligned}\mathbb{E}[\gamma_k C_k^* | Q_k] &= \int_{\frac{N_0 \gamma_k}{PL_{kk}(J'_k(Q_k) - \gamma_k)}}^{\infty} \log_2 \left(\frac{PL_{kk}x}{N_0} \left(\frac{J'_k(Q_k)}{\gamma_k} - 1 \right) \right) e^{-x} dx \\ &= \frac{\gamma_k}{\ln 2} E_1 \left(\frac{N_0 \gamma_k}{PL_{kk}(J'_k(Q_k) - \gamma_k)} \right),\end{aligned}\quad (42)$$

where $E_1(z) \triangleq \int_z^{\infty} \frac{e^{-t}}{t} dt$ is the exponential integral function. Otherwise, $\mathbb{E}[\gamma_k C_k^* | Q_k] = 0$. Similarly, if $J'_k(Q_k) > \gamma_k$, the expected data rate is

$$\begin{aligned}\mathbb{E}[R_k^0(H_{kk}, C_k^*) | Q_k] &= \int_{\frac{N_0 \gamma_k}{PL_{kk}(J'_k(Q_k) - \gamma_k)}}^{\infty} \log_2 \left(\frac{1 + PL_{kk}x/N_0}{1 + \frac{1}{(J'_k(Q_k)/\gamma_k - 1)}} \right) e^{-x} dx \\ &= \frac{e^{\frac{N_0}{PL_{kk}}}}{\ln 2} E_1 \left(\frac{N_0 J'_k(Q_k)}{PL_{kk}(J'_k(Q_k) - \gamma_k)} \right).\end{aligned}\quad (43)$$

Otherwise, $\mathbb{E}[R_k^0(H_{kk}, C_k^*) | Q_k] = 0$.

We then calculate c_k^{∞} . Since (40) should hold when $Q_k = 0$, we have

$$c_k^{\infty} = \mathbb{E}[\gamma_k C_k^* | Q_k = 0] \quad (44)$$

$$\mathbb{E}[R_k^0(H_{kk}, C_k) | Q_k = 0] = \lambda_k. \quad (45)$$

Using (42) and (43), we can calculate c_k^{∞} as shown in Lemma 2. Substituting (42), (43), and c_k^{∞} into (40) and letting $a_k \triangleq \frac{N_0}{PL_{kk}}$, we have the following ODE:

$$\beta_k \frac{Q_k}{\lambda_k} + \frac{\gamma_k}{\ln 2} E_1 \left(\frac{a_k \gamma_k}{J'_k(Q_k) - \gamma_k} \right) + J'_k(Q_k) \lambda_k - J'_k(Q_k) \frac{e^{a_k}}{\ln 2} E_1 \left(\frac{a_k J'_k(Q_k)}{J'_k(Q_k) - \gamma_k} \right) - c_k^{\infty} = 0. \quad (46)$$

According to Section 0.1.7.3 of [20], we can obtain the parametric solution of (46), as shown in (17) in Lemma 2.

APPENDIX E

PROOF OF THEOREM 3

Taking the first order Taylor expansion of the L.H.S. of the Bellman equation in (14) at $L_{ij} = 0$ ($\forall i \neq j$), $C_k = C_k^*$, where C_k^* minimizes the L.H.S. of (40), and using parametric optimization analysis [21], we have the following result regarding the approximation error:

$$J(\mathbf{Q}; \delta) - J(\mathbf{Q}; 0) = \sum_{i=1}^K \sum_{j=1, j \neq i}^K L_{ij} \tilde{J}_{ij}(\mathbf{Q}) + \mathcal{O}(\delta^2), \quad (47)$$

where $\tilde{J}_{ij}(\mathbf{Q})$ captures the coupling terms in $J(\mathbf{Q})$ satisfying

$$\begin{aligned} & \sum_{k=1}^K \left(\lambda_k - \mathbb{E} \left[\log_2 \left(1 + \frac{PL_{kk} \|\tilde{H}_{kk}\|^2}{N_0 + N_k^*} \right) \middle| \mathbf{Q} \right] \right) \frac{\partial \tilde{J}_{ij}(\mathbf{Q})}{\partial Q_k} \\ & + \mathbb{E} \left[\frac{\|\tilde{H}_{ij}\|^2}{\ln 2} \left(\frac{\frac{J'_i(Q_i)}{N_0 + N_i^*}}{1 + \frac{N_0 + N_i^*}{PL_{ii} \|\tilde{H}_{ii}\|^2}} \sum_{l=1, l \neq i}^K \frac{N_0 + N_l^*}{L_{ll} \|\tilde{H}_{ll}\|^2} \right. \right. \\ & \left. \left. + \sum_{k=1, k \neq i, j}^K \frac{\frac{J'_k(Q_k)}{N_0 + N_k^*}}{\left(1 + \frac{N_0 + N_k^*}{PL_{kk} \|\tilde{H}_{kk}\|^2} \right)} \frac{N_0 + N_j^*}{L_{jj} \|\tilde{H}_{jj}\|^2} \right) \middle| \mathbf{Q} \right] = \tilde{\theta}_{ij}, \end{aligned} \quad (48)$$

with boundary condition $\tilde{J}_{ij}(\mathbf{Q})|_{Q_i=0} = 0$ or $\tilde{J}_{ij}(\mathbf{Q})|_{Q_j=0} = 0$, where $N_k^* = \frac{PL_{kk} \|\tilde{H}_{kk}\|^2 + N_0}{2^{C_k^*} - 1}$ and $\tilde{\theta}_{ij} = \frac{\partial \theta}{\partial L_{ij}}$ is constant (where we treat θ as a function of $\{L_{ij} : \forall i \neq j\}$). According to (42), we have

$$\mathbb{E} \left[\log_2 \left(1 + \frac{PL_{kk} \|\tilde{H}_{kk}\|^2}{N_0 + N_k^*} \right) \middle| \mathbf{Q} \right] = \frac{e^{a_k} E_1(a_k)}{\ln 2} \mathcal{O}(1). \quad (49)$$

Then, we calculate the second term in (48) and each part is calculated as follows:

$$\begin{aligned} \mathbb{E} \left[\frac{\frac{J'_i(Q_i)}{N_0 + N_i^*}}{1 + \frac{N_0 + N_i^*}{PL_{ii} \|\tilde{H}_{ii}\|^2}} \middle| \mathbf{Q} \right] &= \mathbb{E} \left[\frac{PL_{ii} \|\tilde{H}_{ii}\|^2}{\left(N_0 + PL_{ii} \|\tilde{H}_{ii}\|^2 \right) N_0} \middle| \mathbf{Q} \right] \mathcal{O}(J'_i(Q_i)) \\ &= \frac{\beta_i}{\lambda_i} \left(1 - \frac{a_i e^{a_i} E_1(a_i)}{N_0} \right) \mathcal{O}(Q_i) \end{aligned} \quad (50)$$

$$\mathbb{E} \left[\frac{N_0 + N_j^*}{L_{jj} \|\tilde{H}_{jj}\|^2} \middle| \mathbf{Q} \right] = \frac{2N_0}{L_{jj}} \mathbb{E} \left[\frac{1}{\|\tilde{H}_{jj}\|^2} \middle| \mathbf{Q} \right] \mathcal{O}(1) = \frac{2N_0}{L_{jj}} \mathcal{O}(1). \quad (51)$$

Substituting these calculation results into (48), we rewrite the PDE as

$$\begin{aligned}
& \sum_{k=1}^K \left(\lambda_k - \frac{e^{a_k} E_1(a_k)}{\ln 2} \mathcal{O}(1) \right) \frac{\partial \tilde{J}_{ij}(\mathbf{Q})}{\partial Q_k} \\
& + \frac{\beta_i}{\lambda_i \ln 2} \left(1 - \frac{a_i e^{a_i} E_1(a_i)}{N_0} \right) \sum_{l=1, l \neq i}^K \frac{2N_0}{L_{ll}} \mathcal{O}(Q_i) \\
& + \sum_{k=1, k \neq i, j}^K \frac{\beta_k}{\lambda_k \ln 2} \left(1 - \frac{a_k e^{a_k} E_1(a_k)}{N_0} \right) \frac{2N_0}{L_{jj}} \mathcal{O}(Q_k) = 0.
\end{aligned} \tag{52}$$

Using 3.8.2.1 of [22] and taking into account the boundary conditions, we obtain that

$$\tilde{J}_{ij}(\mathbf{Q}) = \frac{\frac{\beta_i}{\lambda_i} \left(1 - \frac{a_i e^{a_i} E_1(a_i)}{N_0} \right) \sum_{l=1, l \neq i}^K \frac{N_0}{L_{ll}}}{e^{a_i} E_1(a_i) - \lambda_i \ln 2} \mathcal{O}(Q_i^2) + \frac{N_0}{L_{jj}} \sum_{k=1, k \neq i, j}^K \frac{\frac{\beta_k}{\lambda_k} \left(1 - \frac{a_k e^{a_k} E_1(a_k)}{N_0} \right)}{e^{a_k} E_1(a_k) - \lambda_k \ln 2} \mathcal{O}(Q_k^2). \tag{53}$$

Substituting it into (47) and exchanging the indices i and k , we obtain the first order perturbation (18) in Theorem 3.

APPENDIX F

PROOF OF LEMMA 3

We adopt the following argument to prove the convexity [23]: given two feasible points \mathbf{x}_1 and \mathbf{x}_2 , define $g(t) = f(t\mathbf{x}_1 + (1-t)\mathbf{x}_2)$, $0 \leq t \leq 1$, then $f(\mathbf{x})$ is a convex function of \mathbf{x} if and only if $g(t)$ is a convex function of t , which is equivalent to $\frac{d^2 g(t)}{dt^2} \geq 0$ for $0 \leq t \leq 1$. To use this argument, we rewrite problem (20) as

$$\min_{\mathbf{C}} f(\mathbf{C}, \delta) = \sum_{k=1}^K \left(\gamma_k C_k - \frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} R_k(\mathbf{H}, \mathbf{C}) \right). \tag{54}$$

Consider the convex combination of two feasible solutions, $\mathbf{C}^{(1)} = \{C_k^{(1)} : \forall k\}$ and $\mathbf{C}^{(2)} = \{C_k^{(2)} : \forall k\}$, as $\mathbf{C}^c = \{C_k^c = tC_k^{(1)} + (1-t)C_k^{(2)} : \forall k\}$ and $0 \leq t \leq 1$. When δ is sufficiently small, the second order derivative of $f(\mathbf{C}^c, \delta)$ is calculated as

$$\begin{aligned}
\frac{d^2 f(\mathbf{C}^c, \delta)}{dt^2} = & \sum_{k=1}^K \left(\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} \ln 2 \left(\frac{-P^2 - 2PZ_k}{(P + Z_k)^2 Z_k^2} \left(\sum_{j=1}^K \frac{X_j Y_j \|S_{kj}\|^2 (C_j^{(1)} - C_j^{(2)})^2}{(X_j - 1)^2} \right)^2 \right. \right. \\
& \left. \left. + \frac{P}{(P + Z_k) Z_k} \sum_{j=1}^K \frac{(X_j^2 + X_j) Y_j \|S_{kj}\|^2 (C_j^{(1)} - C_j^{(2)})^2}{(X_j - 1)^3} \right) \right),
\end{aligned} \tag{55}$$

where $X_j = 2^{tC_j^{(1)} + (1-t)C_j^{(2)}}$ and $Z_k = \sum_{j=1}^K \|S_{kj}\|^2 (N_0 + N_j)$. When δ is sufficiently small, the terms in the order of $\mathcal{O}(\delta)$ can be ignored and we simplify $\frac{d^2 f(\mathbf{C}^c, \delta)}{dt^2}$ as

$$\frac{d^2 f(\mathbf{C}^c, \delta)}{dt^2} \approx \sum_{k=1}^K \frac{\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} P X_k^3 Y_k (P N_0 + \|S_{kk}\|^2 N^2) \|S_{kk}\|^4 \left(C_k^{(1)} - C_k^{(2)}\right)^2 \ln 2}{(X_k - 1)^4 (P + Z_k)^2 Z_k^2}. \quad (56)$$

It is obvious that $\frac{d^2 f(\mathbf{C}^c, \delta)}{dt^2} > 0$ in (56), and thus, the problem (20) is a convex optimization problem for sufficiently small δ .

APPENDIX G

PROOF OF THEOREM 4

We prove the convergence by the fictitious game model [16]. We first construct the following capacity-price fictitious game model. The optimization problem of the fictitious capacity player k is

$$\max_{C_k} u_k^{FW} = \frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_k} R_k(\mathbf{H}, \mathbf{C}) - \gamma_k C_k + \sum_{i=1, i \neq k} \pi_{ik} C_k. \quad (57)$$

The optimization problem of the fictitious price player is

$$\max_{\pi_{ik}} u_{ik}^{FC} = - \left(\pi_{ik} - \frac{\partial \left(\frac{\partial \tilde{V}(\mathbf{Q})}{\partial Q_i} R_i(\mathbf{H}, \mathbf{C}) \right)}{\partial C_k} \right)^2. \quad (58)$$

Each player in this game adopts the myopic best response (MBR) to update his strategy. From [24], the MBR updates converge to Nash Equilibrium in the supermodular games, in which the payoff function is supermodular in player i 's strategy and has increasing differences between any component of player k 's strategy and any component of any other player's strategy. Now, we check if this fictitious game is supermodular. It is obvious that each player's payoff function is supermodular in its own one-dimensional strategy. According to the method in [16], we have $\frac{\partial u_k^{FW}}{\partial C_k \partial \pi_{ik}} = 1 > 0, \forall i \neq k$ and the increasing difference condition is satisfied. Therefore, the fictitious game is a supermodular game and always converges.

When δ is sufficiently small, according to Lemma 3, the problem is convex and the supermodular game converges to the unique global optimal solution of the per-stage problem. Furthermore, the approximation error of the priority function in Theorem 3 approaches 0 with

sufficiently small δ . Therefore, the supermodular game converges to the optimal solution of Problem 1 with sufficiently small δ .

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